Relativity Games

Introduction

Relativity is a rather counter intuitive subject. Einstein showed us a lot of what we took for granted as common sense turned out to be false. For example, there is no such thing as universal time, because time flow at different rates for different observers; two events happening simultaneously to one observer may happen at different times to another; the length of an object is relative to the observer; and velocities do not simply add, so if you are running at velocity v and throw out a tennis ball at velocity u (relative to yourself), the tennis ball does not travel at a velocity $u + v$ relative to an observer on the ground.

The fun in relativity is in understanding the logics behind Einstein's relativity. Many of his insights began with attempts to visualize the potential paradoxes that originate from the simple observation that the speed of light c is the same to any inertial observers. In this lab we will try to "reenact" the scenarios that lead to those potential paradoxes, and understand how Einstein resolved those paradoxes using new ideas about time and space.

Reenactment of Thought Experiments

Einstein did most of his "experiments" in his head ("thought experiments"), and many of those thought experiments remain difficult to perform directly even with today's technology, so what we will do in this lab is to pretend that the speed of light c is only $10\ cm/s$. We will use magnets on the white board to denote the photos and see how it moves from the perspective of different reference frames. We will study several scenarios, and at the end of the lab, each group will reenact their scenarios and explain the lesson learned about space and time.

Scenario 1: Relativity of Time

1. We will assume for now that there is such a thing as universal time (i.e., a time that all observers agree on). We will try to deduce the contradiction under such an assumption and how the paradox could be resolved.
2. Have four magnets lined up at time $t = 0s$ as in Figure 1. A fifth magnet will play the role of the photon.
3. Suppose one (universal) second has passed. Remember that $c=10cm/s$, figure out where the photon should be from the point of view of A and have the photon move to that location.
4. B, C and D will now measure their distances from the photon (ignoring the effect of length contraction).
5. Using universal time, what would be the speed of light from the point of view of B, C and D (they will not all be $10cm/s$)?

   c_B = ____________________; c_C = ____________________; c_D = ____________________.

6. Now suppose time is not universal, so that $t_A \neq t_B \neq t_C \neq t_D$, what would the times be for the remaining three observers in order for c to be exactly $10cm/s$?

t_B = ________________; t_C = ________________; t_D = ________________.

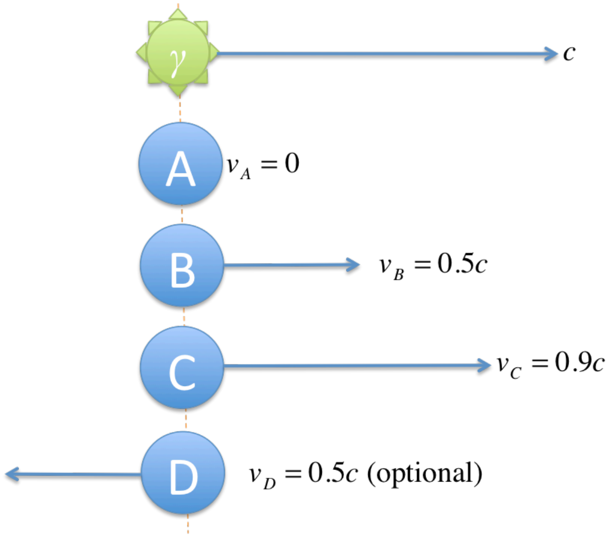

Figure 1: Four observers lined up at *t=0s*.

Warning: While this role-playing game gives insights about the relativity of time, it is an over-simplification that missed some other crucial points of relativity:

(i) One did not take into account the relativity of length.
(ii) The observers in practice cannot measure the position of the photon without having collaborators at the location of the photon to do a *local* measurement, but this leads to other complications such as the synchronization of the collaborator's clock with the observers.

Scenario 2: Synchronization

1. Imagine you are the captain of a starship. You have two clocks at both ends of the ship that you want to reset while you are standing in the middle (Figure 2). We will first look at the situation from the point of view of the starship captain.

Figure 2: An attempt to synchronize the clocks by sending photons.

2. Tape together two to three pieces of paper to form one piece of total length of *70cm*. Draw a rectangle of length *60cm* in the middle as your starship, leaving *5cm* on both sides blank as margins. Draw two clocks at the both ends of the starship.
3. Have a magnet play the role of the captain in the middle and two magnets will be photons.
4. At $t = 0s$, the photons are sent out. A student (the time keeper)will call out the time (*1s, 2s, 3s,* …) and the photons will move accordingly (at $c = 10cm/s$).
5. See when the photons will reach the clocks. You can imagine that the clocks have light detectors so that as soon as they receive the light signal from the captain, they will reset to *0s* and start running.
6. Now picture the same situation from the perspective of an outsider observer. Assume that the starship is moving at a velocity of $v = 0.5c$ relative to the outside observer.
7. Ignoring the effect of length contraction, redo the synchronization. This time as the time keeper call out the time (*1s, 2s, 3s,* …) and everything has to move accordingly (including the photons and the starship).
8. From this perspective, does the synchronization succeed? In other words, do the clocks reset at the same time? If not, which clock has a head start?

Warning: While the effect of length contraction will affect the precise value of time, the basic conclusion remains still the same, that two clocks synchronized to one observer could be out of sync relative to another.

Scenario 3: The Relativity of Simultaneity

1. Imagine you are the captain of a starship standing in the middle of the ship (Figure 3). We will study this scenario from the perspective of an outsider observer. Assume that the starship is moving at a velocity of $v = 0.5c$ relative to the outside observer.

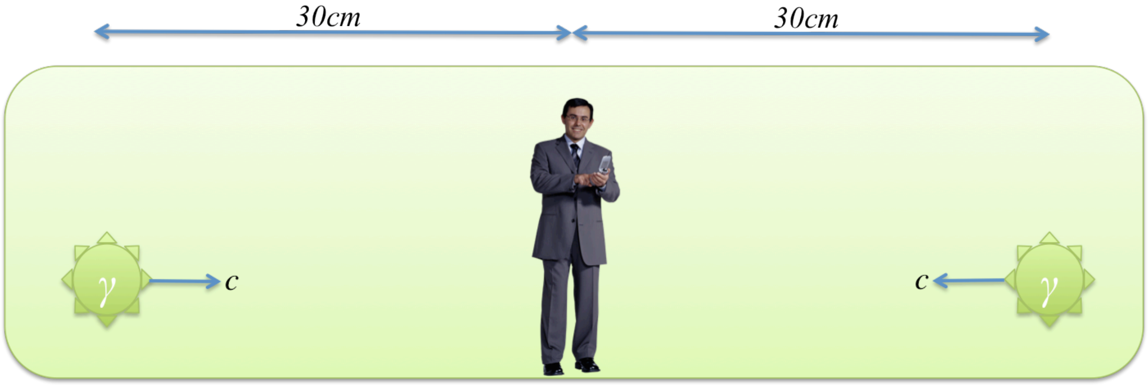

Figure 3: A starship with two simultaneous signals (according to an outside observer) traveling toward the captain.

2. Suppose at $t = 0s$, two asteroids strike the two ends of the ship simultaneously (with respect to the outside observer). As a result, two light signals are generated at both ends to notify the captain.

3. The time keeper will call out the time (*1s, 2s, 3s,* …) and make sure everything moves accordingly (including the starship and the photons).
4. When will the signals reach the captain? Do they arrive at the same time?
5. From the captain's point of view, did the asteroids hit the starship at the same time? If not, which asteroid hit first?

Scenario 4: Length Contraction

1. Imagine you are the captain on a L-shaped starship. You want to send two signals (photons) to your lieutenants at the end of the ship. Your lieutenants will send the photons back as soon as they receive them. Once again, we will first look at the situation from the point of view of the starship captain.

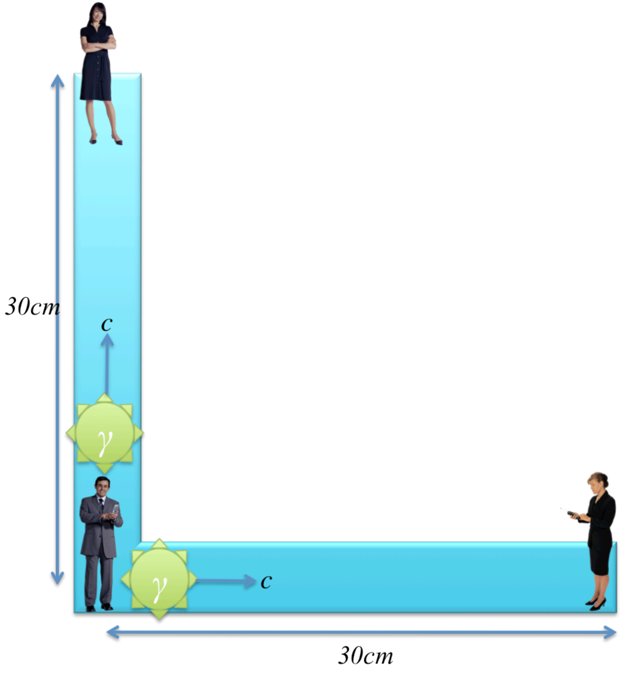

Figure 4: A L-shaped starship.

2. Cut out and tape together paper to form the spaceship. Measure and mark the positions of the captain and the lieutenants *accurately* as in Figure 4.
3. At $t = 0s$, the photons are sent out. The time keeper will call out the time (*1s, 2s, 3s,* …) and the photons will move accordingly (at $c = 10cm/s$).
4. Once the photons reach the lieutenants, they are reflected back toward the captain. Continue the count and see when the photon will reach the captain again.
5. Do the photons get back to the captain at the same time?
6. Now picture the same situation from the perspective of an outsider observer. Assume that the starship is moving at a velocity of $v = 0.5c$ relative to the outside observer.
7. Ignoring the effect of length contraction, redo the steps above. This time a student playing the outside observer will call out the time at <u>half-second</u>

intervals (*0.5s, 1s, 1.5s, 2s,* …) and everything has to move accordingly (including the starship and the photon).

8. The vertical photon has to move very carefully, make sure it does not move more than *5cm* per half-second interval. It should reach the lieutenant at the top around *3.5s*. Theoretically, how long should it take the photon to reach the lieutenant at the top? Show your calculations.
9. When the photons get back to the captain, record the times. Do they arrive at the same time? If not, how can you use length contraction along the direction of motion to resolve the paradox (the paradox is this: since the two photons arrive at the captain at the same time, it must also be true from the perspective of the outside observer)?